R18 Regulation *Subject code:2B4BA*

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A' Grade)

B.Tech IV Semester Supplementary Examinations, July 2022

SPECIAL FUNCTIONS AND COMPLEX VARIABLES

(Common to ECE & EEE)

Maximum Marks: 70 Date:20.07.2022 Duration: 3 hours

Note: 1.This question paper contains two parts A and B.
2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

1 Find the value of $\Gamma(1)$

2 Find the value of $\Gamma(n+1)$,If n is a positive integer

3 If $\varphi(x,y)=x^2-by^2$ is harmonic then find the value of b.

4 Show that analytic function with constant imaginary part is constant.

5 Evaluate $\int_c f(z)dz$ where f(z)=y-x-$3ix^2$ and c is the straight line segment from 0 to 1+i

6 Write Taylor's series expansion of f(z)= $\frac{e^z}{z(z+1)}$ about z=2

7 Define i) Removable singularity
ii) Essential singularity

8 Find the residues of the function f(z)= $\frac{e^{-z}}{(z-2)^4}$

9 Define Bilinear Transformation and give an example

10 Define invariant or fixed points of the Bilinear Transformation.

Part-B

Answer All the following questions. (10M X 5=50Marks)

11 Show that $\beta(m,n)=2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta cos^{2n-1}\theta d\theta$ [10]

OR

12 *Prove that* $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \cdot \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$. [10]

13 If $f(z)=\frac{1}{2}\log\left(x^2+y^2\right)+i\tan^{-1}\left(\frac{px}{y}\right)$ is an analytic function , then find the value of p. [10]

OR

14 Derive the C- R equations in polar form. [10]

15 State and prove Cauchy's integral theorem. [10]

OR

16 Expand the Laurent's series for $f(z)=\frac{z}{(z-1)(z-3)}$ in the region $0<|z-1|<2$ [10]

17 Find the poles and residues at each pole of $f(z)=\frac{\cot z \cdot \cot hz}{(z-1/2)^3}$ on the curve $|z|=1$. [10]

OR

18 Find the poles and the corresponding residues of $\frac{1}{(z^2-1)^3}$ [10]

19 Show that the relation $w=\frac{5-4z}{4z-2}$ transforms the circle $|z|=1$ into a circle of radius unity in the w-plane. [10]

OR

20 Find the bilinear Transformation that maps the points (1-2i,2+i,2+3i) into the points (2+i,1+3i,4). [10]