R22 Regulation *Subject code: 4B3BA*

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A+' Grade)

B.Tech III Semester Regular/Supplementary Examinations, December 2025

COMPLEX ANALYSIS AND VECTOR CALCULUS

(Common to EEE & ECE)

Maximum Marks: 60 Date:15.12.2025 Duration: 3 hours

Note: **1.This question paper contains two parts A and B.**
2. Part A is compulsory which carries 10 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks	(10X1M=10 Marks)	Marks	CO	BTL
1.a)	Show that $u = \frac{x}{x^2+y^2}$ is harmonic.	1M	1	L1
b)	Show that the function $f\ (z)\ =\ xy\ +\ iy$ is everywhere continuous but is not analytic.	1M	1	L1
c)	Expand $f\ (z)\ =\ cosz$ in Taylor's series about $z = \frac{\pi}{4}$.	1M	2	L1
d)	Define a singular point.	1M	2	L1
e)	State the Cauchy residue theorem.	1M	3	L1
f)	What is an improper integral.	1M	3	L1
g)	Estimate $\nabla.(\bar{r})$, where $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}, r = \bar{r}$.	1M	4	L1
h)	Find a unit normal vector to the given surface $x^2y + 2xz = 4$ at the point $(2,-2,3)$.	1M	4	L1
i)	Define line integral on vector point function.	1M	5	L1
j)	State Green's theorem of transformation between line integral and double integral.	1M	5	L1

Part-B

Answer All the following questions.	(5X10M=50Marks)	Marks	CO	BTL
2	Prove that the function defined by $f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z \neq 0 \\ 0, z = 0 \end{cases}$ is continuous and the Cauchy – Riemann equations are satisfied at the origin and $f'(0)$ does not exist.	10M	1	L2
	OR			
3	a) Show that is harmonic $u = e^{-x}(x\ siny - y\ cosy)$ is harmonic. b) Show that xy^2 cannot be real part of an analytic function.	5M 5M	1 1	L2 L2
4	Evaluate the following integrals using Cauchy's integral formulae. (i) $\oint_C \frac{(2z-1)\,dz}{(z-1)(z-2)}\,dz$, where $C: \lvert z \rvert = 3$.	5M	2	L3

	(ii) $\oint_C \frac{e^{-z}\, dz}{(z+1)^3}\, dz$, where $C: \lvert z \rvert = 2$.	5M		
	OR			
5	Expand the function $f(z) = \frac{1}{(z-1)(z-2)}$ in the region by using Taylor's and Laurent's series i) $\lvert z \rvert < 1$ (ii) $1 < \lvert z \rvert < 2$ (iii) $\lvert z \rvert > 2$ (iv) $0 < \lvert z-1 \rvert < 1$.	10M	2	L2
6	a) Evaluate using Residue theorem $\int_C \frac{z^3 dz}{(z-1)^2(z-3)}\, dz$ where C is $\lvert z \rvert = 2$. b) Evaluate using Residue theorem $\int_C \frac{e^z dz}{(z^2+\pi^2)^2}\, dz$ where C is $\lvert z \rvert = 4$.	5M 5M	3 3	L3 L3
	OR			
7	Evaluate $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\, dx$.	10M	3	L3
8	a) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - xz)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential. b) Find the directional derivative of $xyz^2 + xz$ at (1,1,1) in a direction of the normal to the surface $3xy^2 + y = z$ at (0,1,1).	5M 5M	4 4	L2 L2
	OR			
9	Show that $\frac{\bar{r}}{r^3}$ is solenoidal.	10M	4	L3
10	Using Gauss Divergence theorem evaluate $\iint_s \bar{F}.\bar{n}\, ds$, where $\bar{F} = 2x^2y\bar{i} - y^2\bar{j} + 4xz^2\bar{k}$ and s is the closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0, x = 2, y = 0, z = 0$.	10M	5	L3
	OR			
11	Verify the Stoke's theorem for $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$ and surface is the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy - plane.	10M	5	L2

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