*R22 Regulation* *Subject code: 4B3BA*

# TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A+' Grade)

**B.Tech III Semester Supplementary Examinations, July 2024**

## COMPLEX ANALYSIS & VECTOR CALCULUS

***(Common to EEE & ECE)***

***Maximum Marks: 60*** Date:18.07.2024 Duration: 3 hours

**Note:** **1.This question paper contains two parts A and B.**
**2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.**
**3. Part B consists of 5 Units. Answer any one full question from each unit.**
**4. Each question carries 10 marks and may have a, b, c, d as sub questions.**

| | Part-A | CO | Bloom Tx |
|---|---|---|---|
| All the following questions carry equal marks (10X1M=10 Marks) | | | |
| 1.a) | The both real and imaginary part of an analytic function satisfies _____ | CO1 | L1 |
| b) | Which formula can we apply to construct the analytic function whose real part is given? | CO1 | L1 |
| c) | The value of $z\bar{z}$ is _____ | CO2 | L1 |
| d) | If $w = f(z)$ is analytic function, then the derivative of $f(z)$ is_____ | CO2 | L1 |
| e) | A point $z = a$ is called a pole of order n for $f(z)$, if_____ | CO3 | L1 |
| f) | A zero of order one is called _____ | CO3 | L1 |
| g) | The vector $\vec{F} = yz\,\vec{i} + zx\,\vec{j} + xy\vec{k}$ then $\nabla \times \vec{F}$ is_____ | CO4 | L1 |
| h) | If $\varphi$ is a scalar point function, then the magnitude of maximum directional derivative is_____ | CO4 | L1 |
| i) | Gauss divergence theorem states that_____ | CO5 | L1 |
| j) | Which of the theorem relates to surface integral and line integrals? | CO5 | L1 |
| | Part-B | | |
| Answer All the following questions. (5X10M=50Marks) | | | |
| 2 | If f(z) is a regular function of z, Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\|f(z)\|^2 = 4\|f'(z)\|^2$ and also deduce that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = 4\frac{\partial^2}{\partial z\,\partial \bar{z}}$. [10M] | CO1 | L2 |
| | OR | | |
| 3 | State and prove C-R equations in cartesian form. [10M] | CO1 | L2 |
| 4 | Expand $\frac{1}{(z-2)(z-4)}$ as a Taylor series valid in the region $\|z\|=3$. [10M] | CO2 | L2 |
| | OR | | |
| 5 | Using Cauchy's integral formula, evaluate $\int_C \frac{z+4}{z^2+2z+5}dz$ where C is the circle $\|z+1-i\| = 2$. [10M] | CO2 | L2 |
| 6 | Evaluate $\int_C \frac{e^{3z}}{(z-2)(z-3)}dz$, where C is the circle $\|z\|=2$. [10M] | CO3 | L2 |
| | OR | | |

| | | | |
|---|---|---|---|
| 7 | Using contour integration, prove that $\int_0^{2\pi} \frac{d\theta}{a^2-2a\cos\theta+1} = \frac{2\pi}{1-a^2}$, given $a^2 < 1$. [10M] | CO3 | L2 |
| 8 | Find $a$ and $b$ such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at (1,-1, 2). [10M] | CO4 | L2 |
| | OR | | |
| 9 | If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, find $\nabla(\log r)$ also prove that $\nabla^2\left(\frac{1}{r}\right) = 0$. [10M] | CO4 | L2 |
| 10 | Verify Green's theorem in the plane for $\int_c (3x - 8y^2)dx + (4y - 6xy)dy$, where $C$ is the boundary of the region defined by x =0, y=0 and x + y=1. [10M] | CO5 | L2 |
| | OR | | |
| 11 | Verify stokes theorem for $\bar{F} = (y-z)\vec{i} + yz\vec{j} - xz\vec{k}$, where $S$ is the surface bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$ above the XOY plane. [10M] | CO5 | L2 |