



**B.Tech IV Semester Supplementary Examinations, July 2024**

**Special Functions and Complex Variables  
(Common to EEE & ECE)**

**Maximum Marks: 70**

**Date: 18.07.2024 Duration: 3 hours**

- Note:**
1. This question paper contains two parts A and B.
  2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
  3. Part B consists of 5 Units. Answer any one full question from each unit which carries 10M.
  4. Each question carries 10 marks and may have a, b, c, d as sub questions.

**Part-A**

All the following questions carry equal marks		(10X2M=20 Marks)	CO	Bloom Tx
1	Find the value of $\Gamma(1)$		1	L1
2	Solve $\int_0^2 (8 - x^3)^{-1/3} dx$ in terms of Beta function		1	L1
3	If $\phi(x, y) = x^2 - by^2$ is harmonic then find the value of b.		2	L1
4	Show that analytic function with constant imaginary part is constant.		2	L1
5	Evaluate $\int_c^c z^2 dz$ where c is the straight line segment from $O(Z=0)$ to $A(Z=2+i)$ .		3	L1
6	Expand $e^x$ about $z=0$ in Taylor's series expansion		3	L1
7	Find the zeros and its orders of $f(z) = (z - 2)^3 (z - 3)^2$		4	L1
8	Find the residues of $\frac{z.e^z}{(z-1)^3}$ at its poles		4	L1
9	Find the fixed points of the transformation $w = \frac{z-1}{z+1}$ .		5	L1
10	Write two properties of Bilinear Transformation		5	L1

**Part-B**

Answer All the following questions.		(5X10M=50Marks)		
11	Show that a) $\beta(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta d\theta$ <span style="float: right;">[10M]</span> b) $\Gamma(n) = \int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx; n > 0.$		1	L2
OR				
12	Show that a) $\int_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ <span style="float: right;">[10M]</span> b) $\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$		1	L2
13	Find the analytic function whose real part $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 2x + 1$ OR	[10M]	2	L2

14	Prove that the function $f(z)$ defined by $f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$ is continuous and the C-R equations are satisfied at the origin yet $f'(0)$ does not exist. [10M]	2	L2
15	State and prove Cauchy's integral formula. [10M]	3	L2
OR			
16	Evaluate the integrals around $c: z-1 =3$ i) $\int_c \frac{e^z}{(z+1)^2} dz$ ii) $\int_c \frac{e^z}{(z+1)^4} dz$ [10M]	3	L2
17	Evaluate $\int_c \frac{4-3z}{z(z-1)(z-2)}$ ; where $c$ is the circle $ z =\frac{3}{2}$ . using residue theorem [10M]	4	L2
OR			
18	Use the method of contour integration to evaluate $\int_0^\infty \frac{dx}{x^4+a^4}$ [10M]	4	L2
19	S.T the relation $w = \frac{5-4z}{4z-2}$ transforms the circle $ z  = 1$ into a circle of radius unity in the $w$ -plane. [10M]	5	L2
OR			
20	Find the bilinear Transformation that maps the points $(0,i,-1)$ into the points $(0,1,\infty)$ . [10M]	5	L2