



**B.Tech II Year I Semester Supplementary Examinations, July 2024**

**MATHEMATICS IV**

(Common to CE,EEE,ME,ECE,CSE & IT)

**Maximum Marks: 70**

Date:18.07.2024 Duration: 3 hours

- Note:**
1. This question paper contains two parts A and B.
  2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
  3. Part B consists of 5 Units. Answer any one full question from each unit.
  4. Each question carries 10 marks and may have a, b, c, d as sub questions.

**Part-A**

All the following questions carry equal marks (10X2M=20 Marks)

- |    |  |
|----|--|
| 1  | Define Harmonic function.  |
| 2  | Find all values of K, such that $f(z) = e^x(\cos ky + i \sin ky)$ is analytic. |
| 3  | State Cauchy integral theorem.   |
| 4  | State Cauchy integral formula.   |
| 5  | Define essential singularity, removable singularity.                           |
| 6  | Find the residue of $F(Z) = \frac{1}{(Z+1)(Z-1)}$ at $Z= 1$ .                  |
| 7  | If $f(x) = 1$ in $(0,2\pi)$ then find $a_n$ .                                  |
| 8  | If $f(x) = x^2$ in $(-l, l)$ then find $a_0$ .                                 |
| 9  | State the modulation theorem for Fourier transforms.                           |
| 10 | Find the Fourier Sine transform of x.  |

**Part-B**

Answer All the following questions. (5X10M=50Marks)

- |    |  |
|----|--|
| 11 | Show that for $f(z) = \begin{cases} \frac{(x^3-y^3)+i(x^3+y^3)}{x^2+y^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$<br>$f(z)$ does not exist at origin, although C – R equations are satisfied at the origin. [10] |
| OR |  |
| 12 | Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic, find the conjugate function ' V ' and express $u + iv$ as an analytic function of Z. [10]   |
| 13 | State and prove Taylor's theorem. [10]   |
| OR |  |
| 14 | Evaluate: (i) $\oint \frac{e^z}{(z-1)(z-4)} dz$ around the circle $ z  = 2$ . [5]<br>(ii) $\oint \frac{\sin 6z}{(z-\frac{\pi}{2})^3} dz$ around a circle $ z  = 1$ . [5]   |
| 15 | Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^6+1}$ . [10]   |

OR

16	Find the Residue at each pole of $f(z) = \frac{4z^2 - 4z + 1}{(z-1)^2(z-2)(z^2+4)}$ . [10]
17	Find the Fourier series of $f(x) = x - x^2$ in the interval $[-\pi, \pi]$ . [10]
	OR
18	Obtain the Fourier Sine series of $f(x) = \sin x$ in $(0, \pi)$ . [10]
19	Find the finite Fourier sine and Cosine transform, of $f(x) = \sin ax$ in $(0, \pi)$ [10]
	OR
20	Solve the boundary value problem $u_{tt} = a^2 u_{xx}$ ; $0 < x < l$ ; $t > 0$ with $u(0,t) = 0$ ; $u(l,t) = 0$ and $u(x,0) = 0$ , $u_t(x,0) = \sin^3\left(\frac{\pi x}{l}\right)$ [10]