R20 Regulation *Subject code:3B1AA*

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A' Grade)

B.Tech I Semester Supplementary Examinations, April 2023

LINEAR ALGEBRA, CALCULUS & PARTIAL DIFFERENTIAL EQUATIONS

(Civil Engineering)

Maximum Marks: 70 Date:04.04.2023 Duration: 3 hours

Note: 1.This question paper contains two parts A and B.
2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

1 State the conditions to find the consistency of system of non-homogeneous equations.

2 For which value of 'λ' the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ \lambda & 13 & 10 \end{bmatrix}$ is 2.

3 If '2' is an Eigen value of the matrix $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$, find the other two Eigen values.

4 Express the following quadratic form matrix notation
$2x^2 + 3y^2 - 5z^2 - 2xy + 6xz - 10yz.$

5 Find c using Cauchy's mean value theorem for the function $f(x)=e^x(\sin x-\cos x)$ in $[\frac{\pi}{4}\frac{5\pi}{4}]$.

6 Define Beta and Gamma functions.

7 Find the degree of the homogeneous functions $Z = \frac{\sqrt{x}+\sqrt{y}}{x+y}$

8 If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

9 Form a partial differential equation for the equation $z = ax + by + ct$

10 Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$.

Part-B

Answer All the following questions. (5X10M=50Marks)

11 Find the rank of the matrix by reducing to Normal form where: [10M]

$$A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 5 & 2 & 3 & 0 \\ 9 & 8 & 0 & 8 \end{bmatrix}.$$

OR

12 Investigate for what values of k the equations $x + y + z = 1$; $2x + y + 4z = k$; $4x + y + 10z = k^2$ have infinite number of solutions. [10M]

13 Verify Caylay – Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1} and A^4. [10M]

OR

14 Determine the Eigen values and Eigen vectors of the following matrices. [10M]

$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

15 Prove that if $0 < a < 1, 0 < b < 1$ and $a < b$, then $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$ and hence, deduce that $\frac{\pi}{6} - \frac{1}{2\sqrt{3}} < \sin^{-1}\frac{1}{4} < \frac{\pi}{6} - \frac{1}{\sqrt{15}}$. [10M]

OR

16 Derive relation between Beta and Gamma function. [10M]

17 Find the extreme values of $f(x, y) = sinx.siny.\sin(x + y)$. [10M]

OR

18 If $u = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$. [10M]

19 Solve $z(z^2 + xy)(px - qy) = x^4$ [10M]

OR

20 a) Solve $p + q^2 = 1$. [5M]

b) Solve $p(1 + q) = qz$. [5M]