R20 Regulation *Subject code:3B3DA*

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A' Grade)

B.Tech III Semester Regular/Supplementary Examinations, March/April 2023

COMPLEX ANALYSIS & VECTOR CALCULUS

(Electronics and Communication Engineering)

Maximum Marks: 70 Date:27.03.2023 Duration: 3 hours

Note: 1.This question paper contains two parts A and B.
2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

1 Find a function w such that $w = u + i\,v$ is analytic given $u = x^4 - 6x^2y^2 + y^4$

2 Find 'k' such that $f(x,y) = x^3 + 3kxy^2$ is harmonic.

3 Find the Taylor's series expansion of e^z about $z = 3$.

4 What is the Principal part of Laurent's theorem

5 Find the zeros and its orders of $f(z) = (z-2)^3(z-3)^2$

6 Define Contour Integration?

7 Find the normal vector to the surface $x^2y + 2xz = 4$ at the point (2,-2,3).

8 Prove that curl grad $\emptyset = 0$.

9 Define surface integral?

10 What is the Geometrical difference between Stoke's theorem and Green's theorem.

Part-B

Answer All the following questions. (5X10M=50Marks)

11 a) Derive the C- R equations in polar form. 5M

b) Show that $u(x,y) = e^x cosy$ is harmonic and find its harmonic conjugate v(x,y), also the analytic function. 5M

OR

12 a) State and prove Cauchy's integral theorem. 5M

b) If f(z) is a Regular function of z, then prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2$. 5M

13 Find the Laurent's expansion of $f(z) = \frac{1}{z^2-4z+3}$ for $1<|Z|<3$ 10M

OR

14 a) State and prove Cauchy's Residue theorem. 5M

b) Find the poles and residues at each pole of $f(z) = \frac{1-e^z}{z^4}$. 5M

15 Evaluate the integrals $\int_c \frac{z-3}{z^2+2z+5} dz$ where 'c' is the circle given by $|z+1-i|=2$ 10M

OR

16 Use the method of contour integration to evaluate $\int_0^\infty \frac{dx}{x^4+a^4}$ dx. 10M

17 Prove that if $\bar{r}$ is the position vector of any point in space, then $r^n\bar{r}$ is irrotational 10M

OR

18 a) Prove that $div(\emptyset\bar{a}) = (grad\emptyset).\bar{a} + \emptyset\, div\bar{a}$. 5M
b) Prove that $\nabla x(\nabla x\bar{a}) = \nabla(\nabla.\bar{a}) - \nabla^2\bar{a}$. 5M

19 Verify Gauss divergence theorem for $F = x^3\bar{\imath} + y^3\bar{\jmath} + z^3\bar{k}$ taken over the Cube bounded by $x = y = z = 0$ and $x = y = z = a$. 10M

OR

20 Verify Greens theorem in the plane for $\int_c (x^2 - xy^3)\,dx + (y^2\text{-}2xy)\,dy$ where c is a square with vertices (0,0) ,(2,0) ,(2,2) ,(0,2). 10M