



B.Tech II Year I Semester Supplementary Examinations, March/April 2023

Mathematics-IV
(Common to CE,EEE,ME,ECE,CSE &IT)

Maximum Marks: 70

Date:27.03.2023 Duration: 3 hours

- Note:
1. This question paper contains two parts A and B.
 2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
 3. Part B consists of 5 Units. Answer any one full question from each unit.
 4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks

(10x2M=20 Marks)

- 1 Define analytic function.
- 2 Define conjugate Harmonic function.
- 3 State Cauchy's integral theorem.
- 4 Find the lines fractional transformation that maps the points -1, i, 1 onto the points 0, i, ∞.
- 5 Define essential singularity, removable singularity.
- 6 Write working rule to evaluate the integrals of the type $\int_0^{2\pi} F(\cos\theta, \sin\theta) d\theta$
- 7 If $f(x) = 1$ in $(0, 2\pi)$ then find a_n .
- 8 If $f(x) = x^2$ in $(-l, l)$ then find a_0 .
- 9 State the modulation theorem for Fourier transforms.
- 10 Find the Fourier Sine transform of x.

Part-B

Answer All the following questions.

(5X10M=50Marks)

- 11 Test for Analyticity at the origin for $f(z) = \begin{cases} \frac{x^3y(y-ix)}{x^6+y^2} & , Z \neq 0 \\ 0 & , Z = 0 \end{cases}$ (10M)

OR

- 12 Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic, find the conjugate function 'V' and express $u + iv$ as an analytic function of Z. (10M)

- 13 Expand $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region (i) $0 < |z-1| < 1$ (ii) $1 < |z| < 2$ (10M)

OR

- 14 Evaluate: $\oint \frac{e^z}{(z-1)(z-4)} dz$ around the circle $|z| = 2$. (10M)

15 Evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^6+1}$ (10M)

16 Find the Residue at each pole of $f(z) = \frac{4z^2-4z+1}{(z-1)^2(z-2)(z^2+4)}$. (10M)

17 Find the Fourier series of $f(x) = x - x^2$ in the interval $[-\pi, \pi]$ (10M)

18 Find the half-range Fourier Cosine series of the function $f(x)$ given by (10M)

$$f(x) = \begin{cases} \cos x, & \text{for } 0 < x < \pi/2 \\ 0, & \text{for } \pi/2 < x < \pi \end{cases}$$

19 Solve the boundary value problem $u_{tt} = a^2 u_{xx}$; $0 < x < 1$; $t > 0$ with $u(0,t) = 0$; $u(1,t) = 0$ and $u(x,0) = 0$, $u_t(x,0) = \sin^3(\frac{\pi x}{l})$. (10M)

20 Using the Fourier sine transform solve the P.D.E $\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2}$ for $x > 0, t > 0$ under the boundary conditions $v = v_0$ when $x = 0, t > 0$ and the initial condition $v = 0$ when $t = 0, x > 0$. (10M)