R20 Regulation *Subject code: 3B2AI*

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A' Grade)

B.Tech II Semester Regular/Supplementary Examinations, October 2022

DIFFERENTIAL EQUATIONS & VECTOR CALCULUS

(Mechanical Engineering)

Maximum Marks: 70 Date:11.10.2022 Duration: 3 hours

Note: 1.This question paper contains two parts A and B.
2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

1 Find the orthogonal trajectories of the family of curves $y = ax^2$, $'a'$ being a parameter.
2 Test whether the equation $(x+y)^2dx - (y^2 - 2xy - x^2)dy = 0$ is exact.
3 Find the particular integral of $(D-1)^2y = e^x sinx$.
4 Solve $(D^3 - D^2 + 4D - 4)y = e^x$.
5 Eliminate the arbitrary constant m and c from y=mx+c.
6 Eliminate the arbitrary constants $'a'$ and $'b'$ from $z = ax + by + ab$.
7 If $\vec{F} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$, find $div\ curl\ \vec{F}$.
8 Find the unit vector normal to the surface $x^2 + xy + z^2 = 4$ at the point $(1,-1,2)$.
9 Evaluate $\iint_s (yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k})ds$, where S, the surface of the sphere is in the first octant.
10 State Green's theorem.

Part-B

Answer All the following questions. (5X10M=50Marks)

11 Suppose that a population of bacteria grows naturally. Suppose that the initial population is 100 bacteria, and after 1 hour, the population has grown to 120. How large will the population be after 2 hours? How long will it take for the population to reach 1,000,000 bacteria? (10M)

OR

12 Solve: $(1-x^2)\left(\frac{dy}{dx}\right) + 2xy = x\sqrt{(1-x^2)}$. (10M)

13 Solve: $(D^2 - 4D + 4)y = e^{2x} + cos2x$. (10M)

OR

14 Solve: $(D^2 + 4)y = \tan 2x$, by using method of variation of parameter. (10M)

15 Form a partial differential equation by eliminating the arbitrary functions f and g in $z = x^2 f(y) + y^2 g(x)$ (10M)

OR

16 Using Lagrange's method of multipliers, solve $x(y-z)p + y(z-x)q = z(x-y)$ (10M)

17 Prove $\vec{F} = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$ is irrotational and find its scalar potential. (10M)

OR

18 Find 'a' and 'b' such that the surfaces $ax^3 - by^2z = (a+3)x^2$ and $4x^2y - z^3 = 11$ cut orthogonally at $(2, -1, -3)$. (10M)

19 Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ over the rectangular parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0, z = c$. (10M)

OR

20 Using Stoke's theorem for $\vec{F} = (y-z)\vec{i} + yz\vec{j} - xz\vec{k}$ where S is the surface bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ above the xy plane. (10M)