R20 Regulation

Subject code:3B2AA

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY
(Autonomous, Accredited by NAAC with 'A' Grade)

B.Tech II Semester Regular/Supplementary Examinations, October 2022

ORDINARY DIFFERENTIAL EQUATIONS & VECTOR CALCULUS
(Common to CE,CSE,IT,CSE(AI&ML) & CSE(DS))

Maximum Marks: 70 Date:11.10.2022 Duration: 3 hours

Note: 1.This question paper contains two parts A and B.
2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

1 Solve $\frac{dy}{dx}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$

2 Find the orthogonal trajectories of the family of $y=ax$

3 Solve $4y'''+4y''+y'=0$

4 Solve $(4D^2-4D+1)y=100$.

5 Define Cauchy's Euler's equation

6 Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=0$

7 Find a unit normal vector to the given surface $x^2y+2xz=4$ at the point (2,-2,3)

8 Find constants a, b and c if the vector $\bar{f}(2x+3y+az)i+(bx+2y+3z)j+(2x+cy+3z)k$ *at the point* $(2,-2,3)$ is irrotational.

9 If $\bar{F}=xy\,i-z\,j+x^2k$ and c is the curve $x=t^2, y=2t, z=t^2$ from t=0 to t=1, Evaluate $\int_c F\,dr$.

10 State Gauss divergence theorem.

Part-B

Answer All the following questions. (5X10M=50Marks)

11 a) Solve the differential equation $y(xy+e^x)dx-e^xdy=0$ (5M)

b) Solve the differential equation $\frac{dy}{dx}+x\,y=\ y^2e^{\frac{x^2}{2}}\sin x.$ (5M)

OR

12 a) A body kept in air with temperature 25^0C cools from 140^0C *to* 80^0C in 20 minutes. Find whether the body cools down to 35^0C. (5M)

b) If 30 % of radioactive substance disappears in 10 days, how long will it take for 90% of it to disappear? (5M)

13 Solve the differential equation $(D^4 + 2D^2 + 1)y = x^2cos^2x$. (10M)

OR

14 Solve the differential equation $\frac{d^2y}{dx^2} + y = x\cos x$ by the method of variation of parameters. (10M)

15 a) Solve the differential equation$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = logx\ \sin logx$. (6M)

b) Solve the Legendre's equation:

$$(1 + x)^2\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 2\sin(\log(1 + x)). \qquad (4M)$$

OR

16 Solve the simultaneous equations $t\frac{dx}{dt} + y = 0, t\frac{dy}{dt} + x = 0\ given\ x(1) = 1, y(-1) = 0.$ (10M)

17 a) Find the directional derivative of $\emptyset(x, y, z) = x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of the normal to the surface $f(x, y, z) = x\log z - y^2$ at (-1,2,1). (6M)

b) Prove that$\nabla(r^n) = nr^{n-2}\bar{r}$. (4M)

OR

18 a) Find $Div\ \bar{f}$, where $\bar{f} = r^n\bar{r}$. Find n if it is solenoidal. (4M)

b) Show that the vector $(x^2 - yz)\bar{\imath} + (y^2 - zx)\bar{\jmath} = (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential. (6M)

19 Verify Green's theorem in the plane for $\int_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$, where C is a square with vertices (0,0), (2,0), (2,2), (0,2). (10M)

OR

20 Verify Stoke's theorem for $\bar{F} = (y - z + 2)\bar{\imath} + (yz + 4)\bar{\jmath} - xz\bar{k}$, where s is the surface of the cube x=0 , y=0 , z=0 , x=2 , y=2 , z=2 above the xy-plane. (10M)