R20 Regulation *Subject code:* **3B3DA**

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A' Grade)

B.Tech III Semester Regular Examinations, February 2022

COMPLEX ANALYSIS AND VECTOR CALCULUS

(ELECTRONICS & COMMUNICATION ENGINEERING)

Maximum Marks: 70 Date:21.02.2022 Duration: 3 hours

Note: 1.This question paper contains two parts A and B.
2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

1 Define Analytic & Harmonic Functions.
2 Write the Cauchy Riemann Equations in Cartesian and Polar form.
3 Explain about generalized Cauchy integral formula.
4 State Laurent 's Theorem.
5 Write the Cauchy residue theorem?
6 Define pole of order 'm'.
7 Find a unit normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2,-1)
8 If r = xi +yj+zk then evaluate $\nabla^2 r^2$.
9 State Green's theorem.
10 State Stoke's theorem.

Part-B

Answer All the following questions. (5X10M=50Marks)

11 (a) If $f(z)= u + iv = \frac{1}{z}$. Show that the curves $u=c_1$ and $v=c_2$ intersect orthogonally. 5M
(b) Derive the necessary condition for f(z) to be analytic in Cartesian Coordinates. 5M

OR

12 (a) Show that $u(x,y) = x^3-3xy^2$ is harmonic and find its harmonic conjugate. 5M
(b) Find f(z) = u+iv, Given that $u+ v= \frac{Sin2x}{Cosh2y - Cos2x}$ 5M

13 (a) Expand f (z)= cos z about $z=\pi i$ in Taylor's series. 5M
(b) Find the Laurent expansion of $\frac{1}{(z^2 - 4z + 3)}$ for $1 < |z| < 3$. 5M

OR

14 (a) Expand Log z By Taylor's series about Z=1. 4M

(b) Evaluate $\int_c \frac{z-3}{z^2+2z+5} dz$ Where C is $|z+1-i| = 2$ using Cauchy's Integral formula. 6M

15 (a) Find the Poles of $f(z)=\frac{z^2}{(z-1)(z-2)^2}$ and also find the residues at these poles. 4M

(b) Evaluate $\oint_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$ 6M

OR

16 (a) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. 5M

(b) Evaluate $\int_c \frac{z-1}{(z+1)^2(z-2)} dz$ Where C is $|z-i| = 2$ using Cauchy's Residue theorem. 5M

17 (a) Show that $div(r^n \bar{r}) = (n+3)r^{-n}$ 5M

(b) Show that the vector $\bar{F} = (x+3y)\,\bar{i} + (y-3z)\,\bar{j} + (x-2z)\,\bar{k}$ is solenoidal and also find $\bar{F}.curl\bar{F}$ 5M

OR

18 (a) Prove that curl $(\bar{a} \times \bar{b}) = \bar{a}\, div\bar{b} - \bar{b}\, div\, \bar{a} + (\bar{b}.\nabla)\bar{a} - (\bar{a}.\nabla)\bar{b}$ 5M

(b) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ 5M

19 (a) Find the work done in a moving particle in the force field $F = 3x^2i + (2xz-y)j + zk$ along the straight line from (0,0,0) to (2,1,3) 4M

(b) Evaluate $\int_c [x^2\, dx + 2y\, dy - dz]$ by Stoke's theorem where C is the curve $x^2 + y^2 = 4,\ z = 2$ 6M

OR

20 Verify Gauss divergence theorem for $\vec{F} = (x^3 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - x)\vec{k}$ over the rectangular parallelepiped $0 \le x \le a, 0 \le y \le b,\ 0 \le z \le c$ 10M