R18 Regulation *Subject code:* 2B4BA

TKR COLLEGE OF ENGINEERING AND TECHNOLOGY

(Autonomous, Accredited by NAAC with 'A' Grade)

B.Tech IV Semester Regular/Supplementary Examinations, July 2021

SPECIAL FUNCTIONS AND COMPLEX VARIABLES

(Common to EEE & ECE)

Maximum Marks: 70 Date:13.07.2021 Duration: 3 hours

Note: **1.This question paper contains two parts A and B.**
2. Part A is compulsory which carries 20 marks. Answer all questions in Part A.
3. Part B consists of 5 Units. Answer any one full question from each unit which carries 10M.
4. Each question carries 10 marks and may have a, b, c, d as sub questions.

Part-A

All the following questions carry equal marks (10x2M=20 Marks)

1 Define Beta function.

2 Prove that $\Gamma(n) = (n-1)\Gamma(n-1)$

3 Verify $v = sinhx\,cosy$ is harmonic?

4 Write down Cauchy – Riemann equation for an analytic function.

5 Evaluate $\int \frac{dz}{z-4}$, where 'c' is the circle $|z| = 2$.

6 Expand e^z in a Taylor's series about z = 0.

7 Define Isolated singularity.

8 Find the zeros of $\frac{z^3-1}{z^3+1}$.

9 Define translation in conformal mapping.

10 Find the invariant points of the transformation $w = \frac{2z+6}{z+7}$.

Part-B

Answer All the following questions. (10MX 5=50Marks)

11 Define Gamma function and Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. (10M)

OR

12 Prove that $B(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$. (10M)

13 Show that the function $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic and determine its conjugate. (10M)

OR

14 Test the analyticity of the following functions: (10M)
$(i)\ f(z) = e^z \quad (ii) f(z) = z^3 \quad (iii) f(z) = \frac{1}{z}$.

15 Obtain Laurent's series expansion of $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in $(i) 2 < |z| < 3$ $(ii)\ |z| > 3$. (10M)

OR

16 Using Cauchy' integral formula, evaluate $f(z) = \frac{sin\pi z^2 + cos\pi z^2}{(z-1)(z-2)} dz$ where 'C' is the circle $|z| = 3$. (10M)

17 Using Cauchy' residue theorem, evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$, where C is $|z| = 3$. (10M)

OR

18 Using contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{a+bcos\theta}, a > b > 0$. (10M)

19 Find the image of the infinite strips $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. (10M)

OR

20 Obtain the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. (10M)